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DESCRIPTION OF CREEP WITHIN THE FRAMEWORK OF THE FIELD THEORY OF DEFECTS

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The creep laws are described within the framework of the field theory with the use of evolution equations for the density flux of uniformly distributed defects. For the case of uniaxial deformation under constant stress, it is shown that a certain critical stress that has the sense of creep stability limit exists and two deformation regimes can occur, depending on the magnitude of the external load. The unstable-creep rupture time is determined for the system in the case where the stresses are greater than the critical stress and the initial rate exceeds the unstable stationary rate.

Introduction. The necessity of using materials at high temperatures and loads and the production of new materials whose properties depend strongly on external conditions have motivated many experimental and theoretical studies in the field of creep. By creep, deformation processes for which the stress–strain relations contain the time explicitly or in terms of certain operators [1] are meant. Creep is typical of materials of different physical nature (metals, alloys, rocks, plastics, etc.) at any temperatures (from cryogenic temperatures to temperatures close to the melting point). Obviously, the creep laws and physical mechanisms of this phenomenon are different for different materials and different cases of loading.

The physical theories of creep [2–4] that are based on the concept of crystal-lattice defects give deeper insight into the phenomenon and describe many specific features observed. It is assumed in the above-mentioned studies that elementary creep processes in solids at moderate temperatures are due mainly to dislocation displacements. From the viewpoint of physical mesomechanics [5], a deformable solid is a complex hierarchical system in which interacting defect structures of different scale level form upon deformation.

The behavior of the systems of different nature that include many interacting elements has been the subject matter of synergetics [6]. In synergetics, the micro-, meso-, and macrolevels of description of the system are distinguished. On the microscopic level, separate structural elements are studied by specifying their location, velocities, and interactions. On the mesoscopic level, the variables relevant to an ensemble of structural elements are introduced. When the system is described on the macrolevel, the mesoscopic level is assumed to be the initial level, and methods of predicting the onset of macroscopic structures are developed.

The existing physical theories [2–4] study the creep phenomenon within the framework of the microscopic description of a system in which separate noninteracting defects of the material are considered and, in addition, their general contribution to the strain is determined. In the present study, the specific features of creep are analyzed on the mesoscopic level, where a set of interacting defects is considered and its cooperative properties are taken into account. An equation that relates the defect-flux rate to stresses and allows one to investigate the creep phenomenon is obtained within the framework of the field theory of defects, which describes the dynamics of a dislocation ensemble [7, 8]. This equation is used to investigate the specific features of uniaxial deformation, since many results concerning the creep were obtained in experiments on bars in tension.

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1. Dynamic Equations of a Dislocation Ensemble. Panin et al. [7, 8] suggested treating a deformable solid with defects as a mixture of two, elastic and defect, continua. The elastic continuum is a material medium that undergoes elastic distortions caused by external actions and material defects, and the defect continuum is a mechanical field that transmits the interaction of material volumes and is a carrier of energy and momentum.

Within the framework of this model, the dynamic equations of an elastic defect medium can be written in the form

$$B\partial_k I_{ki} = -P_i, \qquad De_{ikl}\partial_k\alpha_{lj} = -B\,\frac{\partial}{\partial t}\,I_{ij} - \sigma_{ij}; \tag{1.1}$$

$$\frac{\partial}{\partial t}\,P_i = \partial_j\sigma_{ji}. \tag{1.2}$$

Here $\alpha_{ij} = e_{ikl}\partial_k\beta_{lj}^{\rm int}$ is the dislocation-density tensor, $I_{ij} = -(\partial/\partial t)\beta_{ij}^{\rm int} - \partial_i V_j^{\rm int}$ is the dislocation flux density tensor, $\sigma_{ij} = C_{ijkl}(\beta_{kl}^{\rm ext} + \beta_{kl}^{\rm int})$ is the effective-stress tensor, $P_i = \rho(V_i^{\rm ext} + V_i^{\rm int})$ is the effective momentum, ρ is the density of the medium, D and B are the constants of the model, C_{ijkl} is the elastic constant tensor, and e_{ikl} is the antisymmetric Levi-Civita tensor. The quantities α_{ij}, I_{ij}, σ_{ij}, and P_i are determined by the components of elastic distortions caused by the external actions $\beta_{kl}^{\rm ext}$ and the material defects $\beta_{kl}^{\rm int}$, the rate of elastic displacements $V_i^{\rm ext}$, and the velocity $V_i^{\rm int}$ due to defect displacements. Supplemented by the geometrical relations of elastic continuum with defects [9] $\partial_k\alpha_{ki} = 0$ and $(\partial/\partial t)\alpha_{ij} = e_{ikl}\partial_k I_{lj}$, Eqs. (1.1) constitute a complete set of dynamic equations of a dislocation ensemble that satisfies the compatibility condition (1.2).

As shown in [7], in the absence of external actions, the internal stresses and the momentum are determined by the Maxwell stress tensor and the density of field-momentum flux:

$$\sigma_{ij}^{\rm int} = D\Big(\alpha_{ki}\alpha_{kj} - \frac{\delta_{ij}}{2}\,\alpha_{kl}\alpha_{kl}\Big) + B\Big(I_{ki}I_{kj} - \frac{\delta_{ij}}{2}\,I_{kl}I_{kl}\Big) + \eta I_{ij},$$

$$\rho V_i^{\rm int} = Be_{ikl}\alpha_{kn}I_{ln}.$$

Here η is the viscosity coefficient and δ_{ij} is the Kronecker symbol. Taking these equalities into account, we write the dynamic equations of a defect ensemble in the form

$$\begin{gathered}\partial_k\alpha_{ki} = 0, \qquad \frac{\partial}{\partial t}\,\alpha_{ij} = e_{ikl}\partial_k I_{lj}, \qquad B\partial_k I_{ki} = -Be_{ikl}\alpha_{kn}I_{ln} - \rho V_i^{\rm ext}, \\ De_{ikl}\,\partial_k\alpha_{lj} = -B\,\frac{\partial}{\partial t}\,I_{ij} - D\Big(\alpha_{ki}\alpha_{kj} - \frac{\delta_{ij}}{2}\,\alpha_{kl}\alpha_{kl}\Big) - B\Big(I_{ki}I_{kj} - \frac{\delta_{ij}}{2}\,I_{kl}I_{kl}\Big) - \eta I_{ij} - \sigma_{ij}^{\rm ext}.\end{gathered} \tag{1.3}$$

This system allows us to investigate the dynamics of the dislocation ensemble for a given external action determined by the quantities $V_i^{\rm ext}$ and $\sigma_{ij}^{\rm ext}$.

2. Dynamic Equations of an Ensemble of Uniformly Distributed Defects. Koneva and Kozlov [10] analyzed the evolution of defect structures and showed that as the strain increases, the chaotic distribution of defects observed at the yield point becomes a sequence of oriented and disoriented defect substructures. In a continual description, the intensities of a field of chaotically distributed defects (α and I) are independent of coordinates and correspond to the uniform distribution of defects. In this case, Eqs. (1.3) take the form

$$Be_{ikl}\alpha_{kn}I_{ln} = -\rho V_i^{\rm ext}, \qquad \frac{\partial}{\partial t}\,\alpha_{ij} = 0,$$

$$B\,\frac{\partial}{\partial t}\,I_{ij} + D\Big(\alpha_{ki}\alpha_{kj} - \frac{\delta_{ij}}{2}\,\alpha_{kl}\alpha_{kl}\Big) + B\Big(I_{ki}I_{kj} - \frac{\delta_{ij}}{2}\,I_{kl}I_{kl}\Big) + \eta I_{ij} + \sigma_{ij}^{\rm ext} = 0.$$

The second equation of the system implies that the density of dislocations does not depend on time when the material defects are distributed uniformly. Setting $\alpha = 0$, we obtain

$$B\frac{\partial}{\partial t}I_{ij}+B\Big(I_{ki}I_{kj}-\frac{\delta_{ij}}{2}I_{kl}I_{kl}\Big)+\eta I_{ij}+\sigma_{ij}^{\text{ext}}=0; \tag{2.1}$$

$$I_{ij}=\frac{\partial}{\partial t}\beta_{ij}^{\text{int}}=-\frac{\partial}{\partial t}\beta_{ij}, \tag{2.2}$$

where β_{ij} is the plastic distortion which determines the defect flux [9]. We confine our analysis to the case of uniaxial loading. In the approximation of engineering theories that establish a relation between strains, stresses, their rates, and time, only the components $I_{11}=-\xi_{11}$ and σ_{11} in Eq. (2.1), whose tensor indices are dropped below, are nonzero. As a result, we obtain the differential equation

$$\frac{d\xi}{dt}=\frac{1}{2}\xi^2-\frac{\eta}{B}\xi+\frac{1}{B}\sigma$$

which relates the plastic-strain rate to stresses. Introducing the dimensionless variables $v=(B/\eta)\xi$, and $\tau=(\eta/B)t$, we write this equation in the form

$$\frac{dv}{d\tau}=\frac{1}{2}v^2-v+S. \tag{2.3}$$

3. Specific Features of Creep at Constant Stress. We assume that the differential equation (2.3) describes the deformation of a solid in creep. We consider the simplest case $\sigma=\text{const}$, which corresponds to the creep at constant stress. Setting the right side of Eq. (2.3) equal to zero, we determine the stationary points at which the strain rate is constant and analyze their stability. The stationarity condition

$$f(v)=v^2/2-v+S=0$$

yields the steady creep rates

$$v_1=p=1+\sqrt{1-2S},\qquad v_2=q=1-\sqrt{1-2S}.$$

Since the quantities η, B, and σ determining S are positive, the inequality $0<q<p$ holds.

Analyzing the diagram of the function $f(v)$ and the phase pattern of the differential equation (2.3), we infer that the stationary state q is stable and the state p is unstable. When the governing parameter S determining the external action tends to 1/2, the stable and unstable states become close; these states coincide for

$$S^*=1/2, \tag{3.1}$$

and disappear simultaneously for $S>1/2$. Thus, S^* is the critical value of the governing parameter. For $S<1/2$, the behavior of the real system described by Eq. (2.3) becomes stable. The system goes to the stationary state $v_2=q$; thereby, the possibility of experimental determination of the unstable stationary state $v_1=p$ is eliminated. For $S>S^*$, an unstable creep regime with an increased rate occurs.

Let us consider the above results in greater detail. When $S<S^*$, the solution of Eq. (2.3) has the form

$$\frac{2}{p-q}\ln\left|\frac{v(\tau)-p}{v(\tau)-q}\right|=\tau+C,$$

where C is an integration constant which is determined from the initial conditions $v(0)=v_0$. As a result, the solution can be written as follows:

$$v(\tau)=\frac{p-q[(v_0-p)/(v_0-q)]\exp[(p-q)\tau/2]}{1-[(v_0-p)/(v_0-q)]\exp[(p-q)\tau/2]}. \tag{3.2}$$

Figure 1 shows the evolution of the strain rate for $S=0.2$. For small τ, the form of the function $v(\tau)$ is determined by the initial value v_0. The following intervals are distinguished: $0<v_0<q$, $q<v_0<p$, and $v_0>p$. Figure 1a shows the curves $v(\tau)$ calculated for the values of v_0 that belong to the first two intervals. For $v_0>p$, the function $v(\tau)$ shown in Fig. 1b has a singularity of the type $1/x$ for $x=0$, where $x=1-[(v_0-p)/(v_0-q)]\exp[(p-q)\tau/2]$. Consequently, the rupture time at which the strain rate tends to infinity is determined from the formula

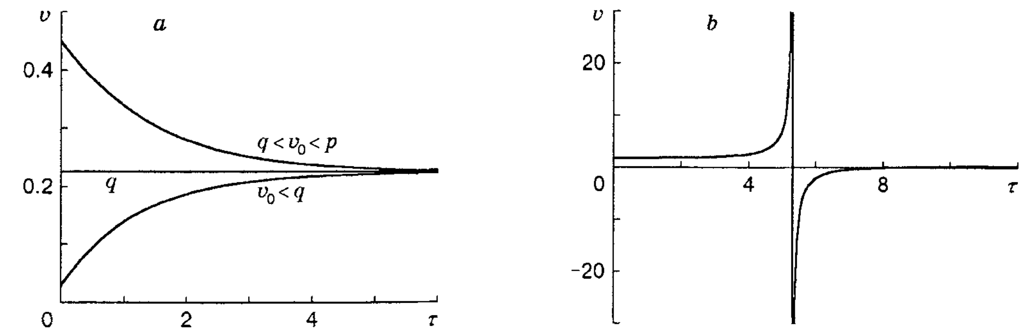

Fig. 1

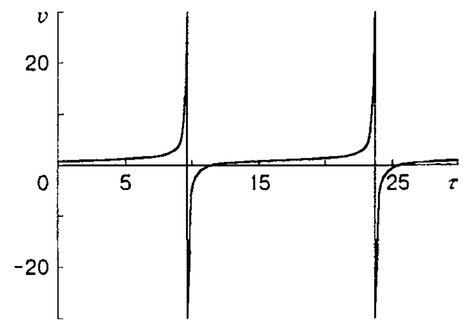

Fig. 2

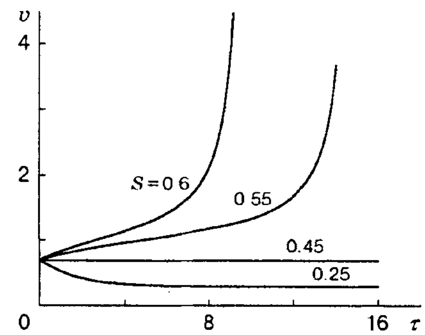

Fig. 3

$$\tau_1 = \frac{2}{p-q} \ln \frac{v_0 - q}{v_0 - p}.$$

The time τ_1 decreases as the external load and the initial loading rate increase. For large values of τ, the strain rate does not depend on v_0; as $\tau \to \infty$, we have $v(\tau) \to q$, i.e., the creep becomes stable.

For $S > S^*$, the solution of Eq. (2.3) can be written as follows:

$$v(\tau) = \frac{n + \alpha^2}{n} + \frac{(n^2 + \alpha^2)\cos(\alpha\tau/2)}{n(\cos(\alpha\tau/2) - (n/\alpha)\sin(\alpha\tau/2))}, \tag{3.3}$$

where $2\alpha = p - q$ and $n = 1 - v_0$. Figure 2 shows the solution (3.3) for $S = 0.6$ and $v_0 = 0.7$. Evidently, the evolution time until the real system disintegrates is restricted by the condition

$$\cos(\alpha\tau/2) - (n/\alpha)\sin(\alpha\tau/2) = 0 \tag{3.4}$$

under which the strain rate tends to infinity. According to (3.4), the "lifetime" of the system before disintegration is $\tau_2 = (2/\alpha)(\arctan(\alpha/n) + \pi)$.

An analysis of the last relations shows that τ_2 decreases as S and v_0 increase. Figure 3 shows the creep strain versus the time for different levels of external load and $v_0 = 0.7$, which satisfies the condition $q(S) < v_0 < p(S)$.

4. Creep Curves. Experimental results are generally represented as a creep curve that characterizes the strain variation with time. Three segments are distinguished on the creep curve [1, 11, 12]. In the first segment, the strain rate gradually decreases to the minimum value, which remains unchanged in the second segment. In the third segment, the strain rate increases, which results in the rupture of a specimen. Within the framework of our approach, the corresponding relations can be obtained by integrating expressions (3.2) and (3.3) over the time. The creep curve is described by the relations

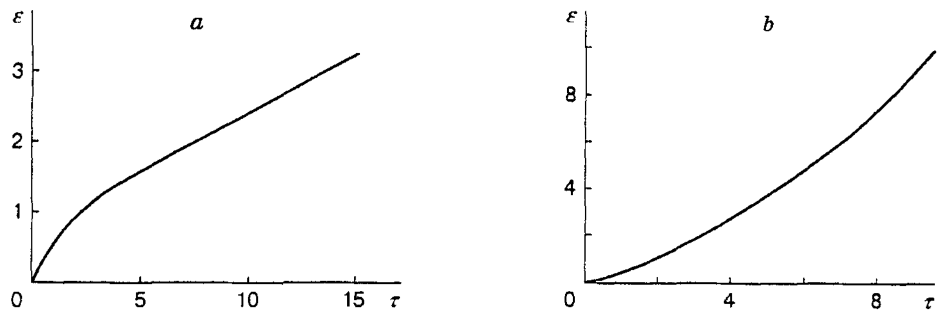

Fig. 4

$$\varepsilon(\tau) = \varepsilon_0 + p\tau + 2\ln |(p-q)/\{p - v_0 - (q - v_0)\exp[(p-q)\tau/2]\}|, \tag{4.1}$$

$$\varepsilon(\tau) = \varepsilon_0 + \tau - 2\ln |\cos(\alpha\tau/2) - (n/\alpha)\sin(\alpha\tau/2)| \tag{4.2}$$

for $S < S^*$ and $S > S^*$, respectively. Here $\varepsilon_0 = \varepsilon(0)$. It follows from (4.1) that $\varepsilon(\tau) \approx C + q\tau$ as $\tau \to \infty$ (C is a constant). This shows that a stationary deformation regime with the constant rate q exists for $S < S^*$. As was noted above, for $S > S^*$, the time before rupture is restricted by condition (3.4). In the limit where τ is small and subject to the condition $\alpha\tau < 1$, expressions (4.1) and (4.2) can be written in the form $\varepsilon(\tau) \approx \varepsilon_0 + v_0\tau$.

Figure 4 shows creep curves calculated for $S = 0.15 < S^*$ and $v_0 = 0.7$ (a) and $S = 0.6 > S^*$ and $v_0 = 0.3$ (b). In both cases, the initial strain was taken to be 0.01%.

5. Discussion of Results. A creep analysis on the basis of the equation that describes the evolution of the flux of uniformly distributed defects shows that the character of the process depends strongly on the external load S and the initial strain rate v_0. For a constant tensile stress, the stable creep region is restricted by the conditions $0 < S < S^*$ and $0 < v_0 < p$, where S^* has the sense of the stable creep limit and is determined, according to (3.1), by the material parameters that describe the inertia of an ensemble of defects and the viscosity of the medium. As follows from the strain-rate evolution analysis, this quantity increases for small τ and $0 < v_0 < q$ and decreases for $q < v_0 < p$ to the minimum stationary value $q(S)$ for $S < S^*$. By virtue of the fact that the creep rate in the first segment of the experimental curves gradually decreases to the minimum rate corresponding to steady creep, one should use $v_0 > q$ as the initial values of $v(\tau)$.

The resulting expressions for $v(\tau)$ agree with the well-known fact that the creep rate increases with stresses [$v_1(S_1) > v_2(S_2)$ for $S_1 > S_2$] [12] and also describe the condition $q(S) = 0$ for $S = 0$ taken into account when this quantity is determined within the framework of phenomenological theories [11].

For $S < S^*$, the stages of unsteady and steady creep can be identified on the creep curve [1, 11, 12]. The above relation for $\varepsilon(\tau)$ is not valid for the third segment of the creep diagram, where the strain rate increases and the deformation terminates with rupture of the specimen. However, the experimental creep curves were obtained at constant load. Rabotnov [1] and Kachanov [12] consider that the accelerated creep is absent up to the moment of specimen rupture in the case of constant stress considered.

Kachanov [12] described the creep curve for $S > S^*$. Pointing out the diversity of creep relations, he considers that the first segment, where the strain rate decreases, can be absent on the creep curve; after a short period of almost constant rate, the creep rate increases, i.e., the diagram contains only the third segment. An analysis of creep within the framework of the field theory of defects shows that the different creep regimes observed in reality occur in the specimen at different levels of applied load.

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